

Critical Points of Correlated Percolation in a Gravitational Link-adding Network Model

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Motivated by the importance of geometric information in real systems, a new model for long-range correlated percolation in link-adding networks is proposed with the connecting probability decaying with a power-law of the distance on the two-dimensional(2D) plane. By overlapping it with Achlioptas process, it serves as a gravity model which can be tuned to facilitate or inhibit the network percolation in a generic view, cover a broad range of thresholds. Moreover, it yields a set of new scaling relations. In the present work, we develop an approach to determine critical points for them by simulating the temporal evolutions of type-I, type-II and type-III links(chosen from both inter-cluster links, an intra-cluster link compared with an inter-cluster one, and both intra-cluster ones, respectively) and corresponding average lengths. Numerical results have revealed objective competition between fractions, average lengths of three types of links, verified the balance happened at critical points. The variation of decay exponents a or transmission radius R always shifts the temporal pace of the evolution, while the steady average lengths and the fractions of links always keep unchanged just as the values in Achlioptas process. Strategy with maximum gravity can keep steady average length, while that with minimum one can surpass it. Without the confinement of transmission range, $\bar{l} \rightarrow \infty$ in thermodynamic limit, while \bar{l} does not when with it. However, both mechanisms support critical points. In two-dimensional free space, the relevance of correlated percolation in link-adding process is verified by validation of new scaling relations with various exponent a , which violates the scaling law of Weinrib's.

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I. INTRODUCTION

Correlated percolation[1–14] is a useful theoretic model in statistical physics. It provides us with fundamental understanding of spread processes of message, disease and matter in nature and society. Linking probability between any two nodes in it takes the form of $p(r) \sim r^{-a}$, where r is d -dimensional distance between the nodes, and a is a positive real number, namely, distance-decay exponent of links. Weinrib and Halperin[15] analytically studied whether the correlations change the percolation behavior or not. Weinrib[16] pointed out, for $a < d$, the correlations are relevant if $a\nu - 2 < 0$, where ν is the percolation-length exponent for uncorrelated percolation; while for $a > d$ the correlations are relevant if $d\nu - 2 < 0$.

It is a generalization of the Harris criterion[17] appears earlier. Recently, network models referring to correlated percolation have gradually appeared[11].

Achlioptas process(AP)[18] for link-adding networks, which is an attractive topic at present[19–38], could be viewed as a new kind of correlated percolation if we put all nodes uniformly on a two-dimensional(2D) plane. Starting from a set of isolated nodes, two candidate links are put to nodes randomly at every time step, but only the link with smaller product $m_i m_j$ is retained, where m_i (or m_j) is the mass (the number of nodes) of the cluster that node i (or j) belongs to, which is called Product Rule(PR). A link chosen with PR from both inter-cluster candidates is called a type-I link. When two candidate-links are of different types, i.e., one is an inter-cluster

link, the other is an intra-cluster one, always the later is retained, and it is called a type-II link. While for both intra-cluster ones, the retained link is arbitrarily chosen no matter they are in the same or different clusters, and it is called a type-III link. Generally speaking, in the way of AP, network percolation is inhibited, which postpones the appearance of the threshold T_c at which a giant component G starts to grow, and results in a sharp growth of G called an explosive percolation. In our point of view, if we put AP on a 2D plane, it gives rise to a new mechanism of long-range correlation for the nodes based on co-evolutionary growing masses of components they connected. The selective rule for topological links relies on mass-product instead of 2D geometric length of them, which prevents the property exhibited in the previous correlated percolation. And correlation feature in AP-type of percolation has not been revealed up till now.

A recent model of ours[39] based on the observation of phenomena in different real systems[40–45] describes another kind of correlated percolation in growing networks, which could be viewed as a overlapping of traditional correlated percolation with AP in a 2D space. The link-occupation function in the model takes the form $p(r) \sim m_i m_j / r_{ij}^a$ which looks like Newton's gravity rule. It resumes the classical Erdos-Renyi(ER) random graph model when exponent $a \rightarrow \infty$, and it gives another extreme of AP when $a \rightarrow 0$. Different properties of such kind of new correlated percolation are expected, since ER random graph grows without any bias, AP takes a strong bias to inhibit network percolation independent of geometric distance, while the new model with gravity-like rule have some distance-related relax on such bias, which produces a new type of correlated percolation.

In this paper, we report the simulation results on objective competition between type-I, II and III links in both gravity model[43, 45] and AP model, and we point out a new mechanism to support critical points, which bears the scaling relations revealed in our recent work. Different saturation effect is manifested, which distinguishes it from the traditional correlated percolations.

II. MODEL

Suppose N isolated nodes are uniformly scattered on a 2D plane. For convenience of calculating distance, the plane is discretized with a triangular lattice, each minimal edge with the length of two units for the convenience of algorithm. Each vertex of the triangles is occupied by a node so that we exclude all possible biases except link-adding rules. For any two pairs of nodes i and j possibly with the same product $m_i m_j$, a type-I link connects the the pair with longer distance if both the links' ends hit the nodes belonging to different clusters; while a type-II link connects the nodes inside the same cluster if the other one is an inter-cluster link; a type-III one connects arbitrarily chosen pair of nodes if both candidate links are intra-cluster ones.

Parallel to PR, we pick randomly two pairs $[(i, j)$ and $(k, l)]$ of nodes in the plane at every time step. For the pair (i, j) (and for (k, l) likewise), we calculate the generalized gravity defined by $g_{ij} \equiv m_i m_j / r_{ij}^a$, where r_{ij} is the geometric distance between i and j , and a is an adjustable decay exponent. Once we have g_{ij} and g_{kl} , we have two choices in selecting which pair to connect. For the case of the maximum gravity strategy (we call it G_{\max}) we connect the pair with the larger value of the gravity, e.g., the link (i, j) is made if $g_{ij} > g_{kl}$ and the link (k, l) otherwise. We also use the minimum gravity strategy (G_{\min}) in which we favor the smaller gravity pair to make connection. The two strategies, G_{\max} and G_{\min} , lead the link-adding networks to evolve along the opposite paths of percolation processes. Generally speaking, G_{\max} facilitates the percolation process, whereas G_{\min} inhibits it. All such generalized gravity values are calculated inside the circular transmission range with the radius R centered at one of nodes i and j as the speaking node [46] in a mobile ad hoc network[41, 42, 45]. For the different limits of parameters R and d , we have three cases in the model. Case I: With the transmission range $R \rightarrow \infty$, we have a generalized gravitation rule which is an extension [45] of widely used gravitation model [43] ($a = 1$) with the tunable decaying exponent a . Case II: With the exponent $a = 0$, we assume that node pairs can be linked with PR topologically inside the transmission range with a limited radius R . Case III: With both limited values of radius R and exponent a , we have the gravity rule inside the transmission range. It can describe the communication or traffics with constrained power or resources.

For case I and case III in the model, three scaling relations have been found with large scale simulations. When strategy G_{\max} is adopted in 2D free space(case I), we have

$$C \sim a^{-\alpha} f(ta^\epsilon) \quad (1)$$

where $t = (T - T_0)/T_0$ is dimensionless time-step with $T_0 = 0.78$, a is the decay exponent of connection probability. $\alpha = 0.01$, $\epsilon = 0.20$, and $f(x)$ is a universal function. When strategy G_{\min} is adopted inside the transmission range with radius R (case III), we have

$$C \sim (a/a_0)^{-\theta} h[t(a/a_0)^\phi] \quad (2)$$

for certain parameter ranges of a and R , where $t = (T - T_0)/T_0$, $T_0 = 1.0$, $\theta = 0.005$, $\phi = -0.50$, $a_0 = 0.5$, and $h(x)$ is a universal function. In addition, when strategy G_{\max} for case III is adopted inside transmission range defined by R , we have another scaling relation

$$C \sim R^{-\delta} H(t\rho^\eta) \quad (3)$$

for $R > 3$, where $\rho = (R - R_0)/R_0$, $R_0 = 2$, $\eta = -0.10$, $\delta = -0.005$, $T_0 = 1.0$ and $H(x)$ is a universal function.

To understand three scaling relations above, we should look into the mechanism of the evolution processes underlying $C(T)$. To see what happens in such critical points

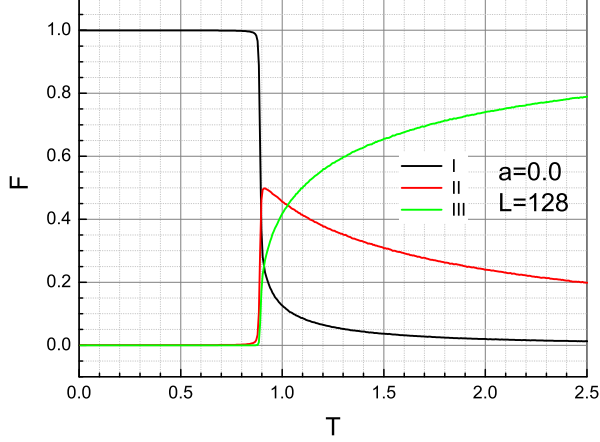


FIG. 1: (color online) Evolution of fractions of type-I, type-II and type-III links in Achlioptas process on 2D plane.

T_0 , and what are particular of them in certain link-adding processes, we count the temporal link fractions $F(T)$, and calculate the average lengths \bar{l} of links which is defined as the summation of all lengths of links for a certain type over its number in a window $\Delta T = 20$ time-steps. By observation of the time-dependent behaviors of fractions of type-I, II and III links, new properties were found out for our gravity-like model together with AP producing explosive percolations.

III. 3. SIMULATION RESULTS

All simulations are carried out on the triangular lattice of the size $N = L \times L$ with $L = 32, 64, 128$ and 256 , respectively. We simulate either of strategy G_{\max} or G_{\min} for either case I or III. The total number of links equating to that of time-steps is divided by N , which is defined as T . The mass of the largest component divided by N makes up the observable C , the node fraction of the largest component. All results presented in this work are obtained from 5000 different realizations of network configurations with $L = 128$ if not specially indicated.

Inspired by Cho and Kahng's [33] work and a referee of ref.[39], we have gone further by calculating fractions of three types of links and arithmetic average lengths of their links. Our attention was pointed at AP first. In Fig.1 we illustrated the evolution of fractions of 3 types of links. Just at the threshold the fraction of type-I (F_I) links has a sharp drop-down, meanwhile that of type-II (F_{II}) shoots up, crossing F_I at $T_c = 0.888$. A little after it, F_I crosses with growing fraction of type-III links (F_{III}) at the level $F_I = F_{III} = 0.25$, while F_{II} gets its summit ($F_{II} = 0.5$) at the same point, which has not been concerned by previous works. However, it is this property that pervades all cases in the present correlated percola-

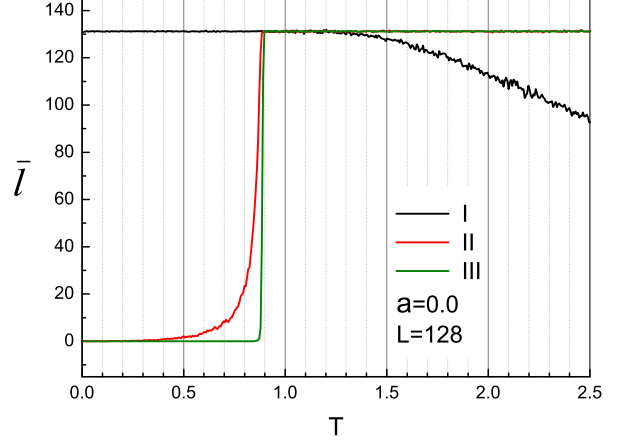


FIG. 2: (color online) Evolution of average lengths of type-I, type-II and type-III links in Achlioptas process on 2D plane.

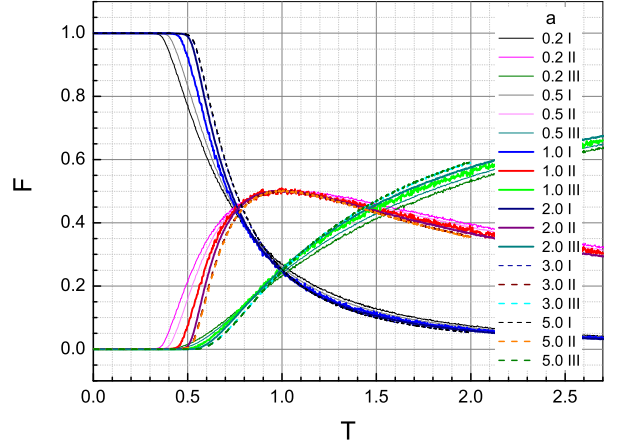


FIG. 3: (color online) Evolution of fractions of type-I, type-II and type-III links with G_{\max} and probability decay exponent $a = 0.2, 0.5, 1.0, 2.0, 3.0$ and 5.0 in case I ($R \rightarrow \infty$).

tion. In Fig.2, the average lengths of type-II (\bar{l}_{II}) merges that of type-III (\bar{l}_{III}) at T_c after an abrupt growth, \bar{l}_{II} starts to grow earlier than \bar{l}_{III} . The level of \bar{l} for both of them keep invariant for $T > T_c$, while \bar{l}_I starts to decrease from T_c . We see from both the figures that in explosive percolation the system undergoes a sharp transition from a type-I link dominant phase into a type-II and III dominant phase at T_c [33]. Besides, average lengths undergo a parallel transition at the same point. Actually, 3 levels of \bar{l} go to infinity in dynamic limit from finite size scaling transformation(not shown).

Now we turn to the fraction of 3 types of links in case I of the present gravity model[39]. With strategy G_{\max} in free 2D space, we have scaling relation (1) for distance-

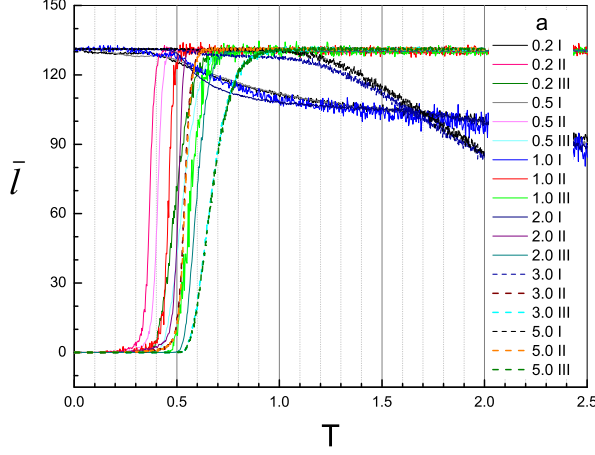


FIG. 4: (color online) Evolution of average lengths \bar{l} of type-I, type-II and type-III links with G_{max} and probability decay exponent $a = 0.2, 0.5, 1.0, 2.0, 3.0$ and 5.0 in case I.

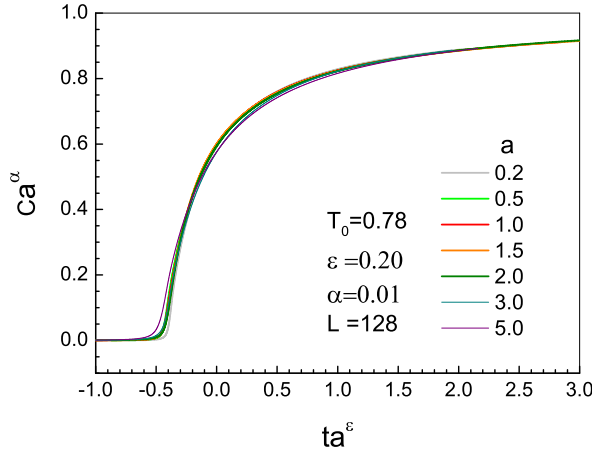


FIG. 5: (color online) Node fraction $C(T)$ of the largest component with strategy G_{max} and the same parameters in Fig. 3.

decay exponent $a \in [0.2, 2.0]$. Fig. 3 shows the evolution of fractions corresponding to it. Curves for F_I cross those of F_{II} with $a = 0.5, 1.0$ and 2.0 around $T = 0.78$ quite clearly, even those with $a = 0.2$ barely cross near it. But fractions for $a = 3.0$ and 5.0 shift the cross point rightward obviously. On the other hand, the curves for F_I cross F_{III} at $T = 1.0$ for almost all values a except $a = 0.2$. To determine which one would be the candidate of another critical point, we cast ourselves on the assistance from the observation of average lengths of links. In Fig. 4 $\bar{l}_{II}(T)$ merges $\bar{l}_{III}(T)$ for $a = 3.0$ and 5.0 at $T = 1.0$, separating themselves from others. However, almost all other ones collect at $T = 0.78$, which gives hint to us for T_0 . (Here a better resolution is needed in further cal-

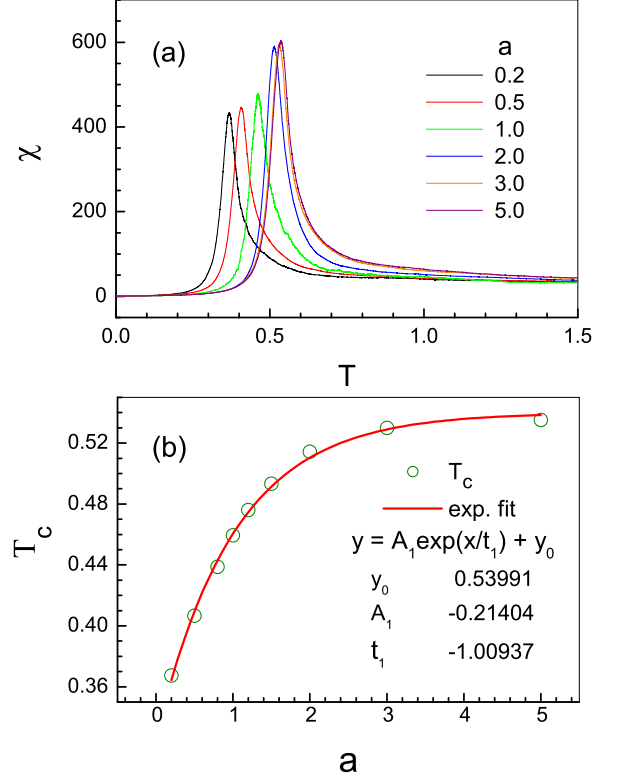


FIG. 6: (color online) (a) Susceptibility $\chi(T)$ of the system with G_{max} in Case I. $L = 32, 64, 128$ and 256 . (b) Percolation thresholds T_c for $a = 0.2, 0.5, 0.8, 1.2, 2.0, 3.0$ and 5.0 with G_{max} in Case I.

ulation). Correspondingly, in Fig. 5 for scaling relation (1) with exponents $\epsilon = 0.2$, $\alpha = 0.01$, and $T_0 = 0.78$, $C(T)$ for $\alpha \in [0.5, 2.0]$ collapse into the universal function very well, with that for $a = 0.2$ barely collapsing onto it. But those $C(T)$ for $a = 3.0$ and 5.0 do not behave well in collapse. The separation from others at the turning middle part indicates the deviation of their T_0 from 0.78 with which others share. In the description of the average lengths for case I with G_{max} , simulated results $\bar{l}(T)$ in Fig. 4 with all values a demonstrate the same steady level ($\bar{l} \simeq 131.50$ for $L = 128$). Variation of parameter a only shifts starting points of up-growing F_{II} and F_{III} as a increases. The saturation effect of large decay exponents ($a = 3.0, 5.0$) appears clearly and is shown by dash lines, which demonstrates the inheritance from traditional correlated percolation [16]. In this case with G_{max} , special level of fractions at cross point of $F_I(T)$ and $F_{III}(T)$ keeps 0.25 just as in AP without any distance-decay included, so does F_{II} at hiking its summit.

As in a usual way, we determine the critical point T_c of percolation by observation of tips of susceptibility $\chi(T)$ (Fig. 6). Comparing T_c in Fig. 6 with $\bar{l}(T)$ in Fig. 4, we find that these T_c approximately hit the horizontal co-

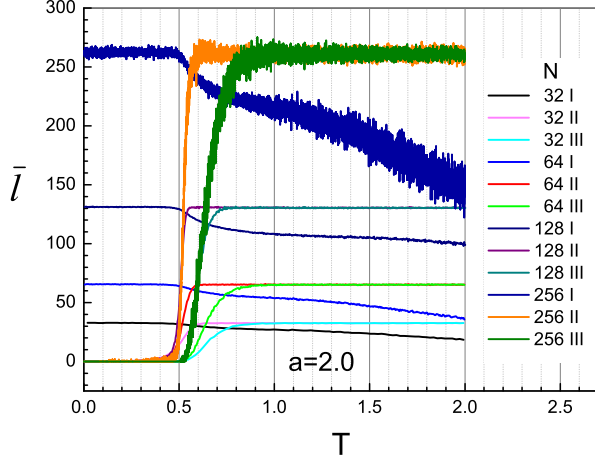


FIG. 7: (color online) Size-dependent average lengths \bar{l} of type-I, type-II and type-III links with strategy G_{\max} in case I. Parameters are the same as in Fig.6

ordinates of middle point of growing fraction of type-II links, which means that T_c is the transition point from the inter-cluster-link dominant phase to the intra-cluster-link dominant phase. Besides T_c , we have another (sub)-critical point T_0 which is in certain range independent of decay exponent a in gravity model, and T_0 indicates the balance between the fractions of type-I and type-II links, yielding a new scaling behavior of $C(T, a)$ in formula (1) not revealed by previous works. Moreover, the steady level $\bar{l} = 131.5$ is always a -independent, type-independent which takes the inherited value of that in AP. Actually, 131.5 is the value for $L=128$ only. We have size effect since a free boundary condition instead of a periodic one is adopted. The finite size effect is shown in Fig.7 which gives that $\bar{l} \sim L$, i.e.,

$$\bar{l} \sim N^{1/2} \quad (4)$$

where N is the number of nodes on the 2D plane. Hopefully it goes towards infinity in thermodynamic limit. However, the finite size effect of $C(T, N)$ (Fig.8 for an example) is not strong enough for us to identify the scaling exponents ν, β as usual. Therefore, we can check the validation of scaling laws presented by Weinrib [16] for correlated percolation in the present model only by rescaling susceptibility $\chi(a, T)$. Fig. 9 illustrates the results of it for examples $a = 0.5$ and 2.0 , respectively. We have scaling relation

$$\chi \sim N^{\gamma/\nu} G((T - T_c)N^{1/\nu}) \quad (5)$$

where $1/\nu = 0.3$, $\gamma/\nu = 0.75$, and $G(x)$ is a universal function. With these values of scaling exponents, the scaling law $\nu_{long} = 2/a$ is not applicable to the present

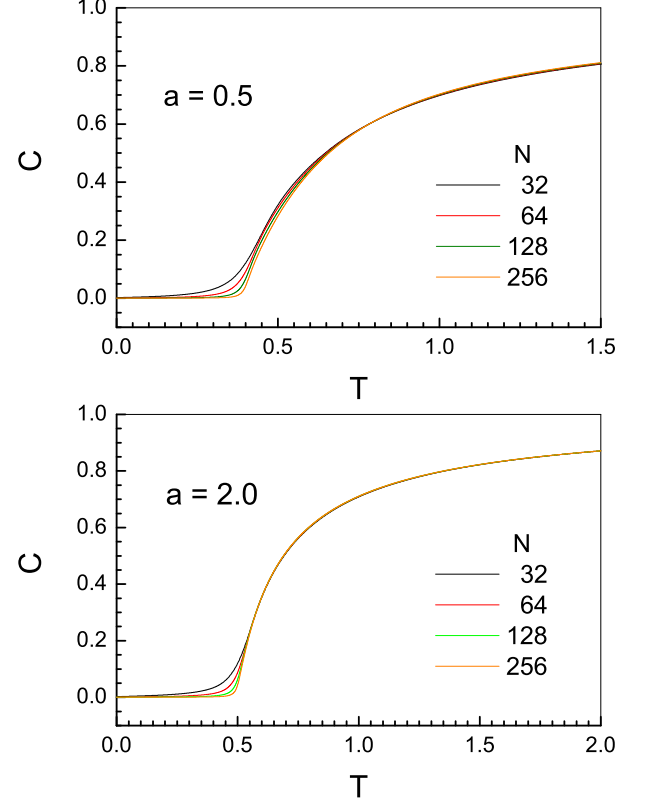


FIG. 8: (color online) Node fraction $C(T)$ of the largest component with strategy G_{\max} in Case I. (a) $a = 0.5$; (b) $a = 2.0$. $N = 32, 64, 128$ and 256 .

model, where ν_{long} is correlation-length exponent in the long-range case.

With power-law form for the correlation function $g(r)$, Weinrib had derived the extended Harris criterion: the long-range nature of the correlations is relevant if $a\nu - 2 < 0$, which means the correlations change the percolation critical behavior. It has been violated since now they all behave differently from traditional short range percolation in a 2D triangular lattice ($\nu = 4/3$) and the correlations are relevant no matter $a\nu - 2$ is less (Fig.9(a)) or larger (Fig.9b) than zero. This is because in strategy G_{\max} we have overlapped the power-law correlation function $g(r)$ with AP which is another kind of autocorrelation process with positive feed back effect of mass-growing.

For the strategy G_{\min} which prefers smaller gravity, it tends to retain a longer link under the comparison of the same product of masses $m_i m_j$. That is to say, long range links have predominance. Evolution of $F_I(T)$ and $F_{II}(T)$ links do not cross at any common point. In Fig.10, the cross points for F_I and F_{II} shift leftward from T_c of AP as decay exponent a increases, which means that G_{\min} as a correlated percolation mechanism weaken the explosive effect caused by AP. But the starting position of fraction-

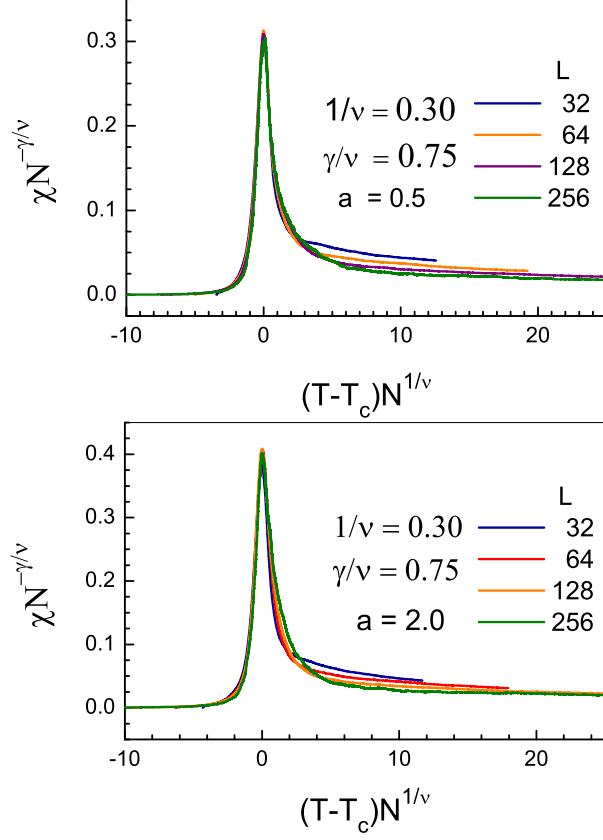


FIG. 9: (color online) Scaling of susceptibility $\chi(T)$ in case I for (a) $a = 0.5$; (b) $a = 2.0$. $N = 32, 64, 128$ and 256 .

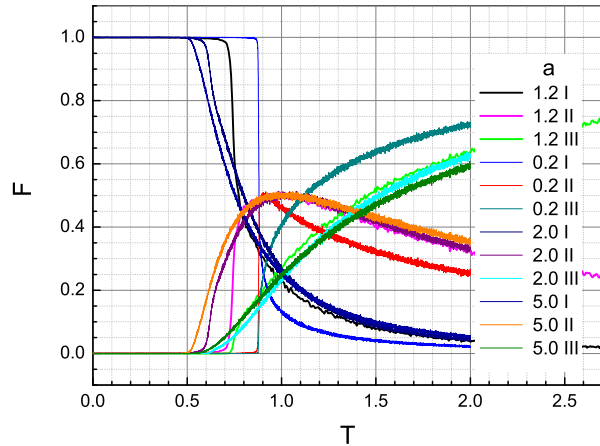


FIG. 10: (color online) Evolution of fractions of type-I, type-II and type-III links with G_{min} and probability decay exponent $a = 0.2, 1.2, 2.0$ and 5.0 in case I. $L = 256$.

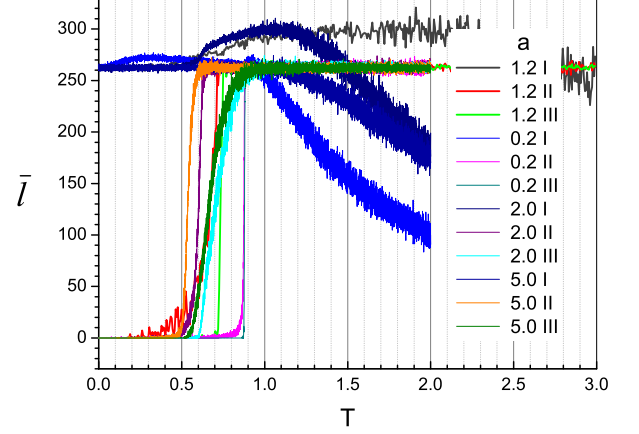


FIG. 11: (color online) Evolution of average lengths of type-I, type-II and type-III links with G_{min} and probability decay exponent $a = 0.2, 1.2, 2.0$ and 5.0 in case I. $L = 256$.

II still provides hints of thresholds T_c . However, it is hard to locate a common cross point for F_I and F_{III} in a range of a . Therefore, we have no scaling relation for them. The steady average lengths of links (Fig.11) have the same level of G_{max} and AP cases, and size-effect (not shown) also tells the divergence of \bar{l} , but all of them do not tell any possible hint for critical points.

Generally speaking, strategy G_{min} facilitates longer links for certain geometric distribution of clusters or nodes. In the evolution, strategy G_{min} emphasis the assignments for different types of links, encourage longer and intra-cluster links. Humps above the steady F_{II} and F_{III} implies out-of-pace growing of link lengths of F_I . That is, F_I surpasses the growing speed of the giant component. Here, it is the geometric distance-dependent strategy that makes G_{min} alleviate effect of AP. With smaller a (e.g., $a=0.2$ in Fig.11) the strategy has the opportunity to exhaust long links before T_c ; while with middle values of a (e.g., $a = 1.2$ and 2.0) it may take longer time to exhaust them. However, with too large a (e.g., $a = 5.0$), G_{min} fall off quicker than the natural dimension, we can only see the AP-type short-range effect of saturation. In this limit, i.e., $a \leq 3.0$, percolations are no longer relevant, which causes saturation of curves $C(T)$ in Fig.1b [39]. However, we should not expect the short-range percolation exponent $\nu = 4/3$ of correlation length here for 2D triangular lattice, since AP has been included in the strategy G_{min} .

The distinct feature in case III for both G_{min} and G_{max} is that the candidate links are selected not only by comparing gravities, but also constrained inside a transmission range R ($r = 2R$ in geometric distance), which ruins the effect comes from the divergence of average link lengths. It is well known that all possible singularities at critical points come from the singularity of correlation length. However, here no length could goes to infinity in

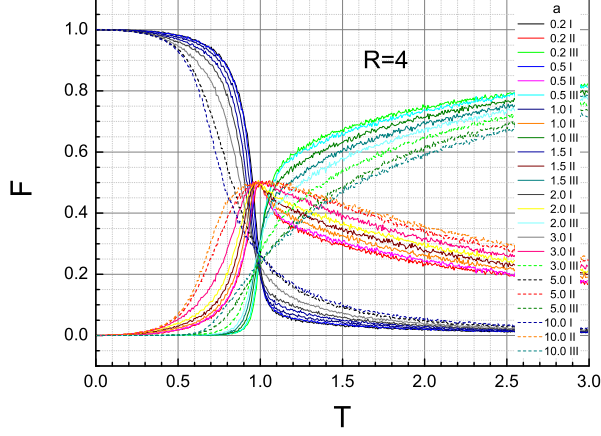


FIG. 12: (color online) Evolution of fractions of type-I, type-II and type-III links with strategy G_{min} and probability decay exponent $a = 0.2, 0.5, 1.0, 1.5, 2.0, 3.0, 5.0$ and 10.0 in case III. $R = 4$.

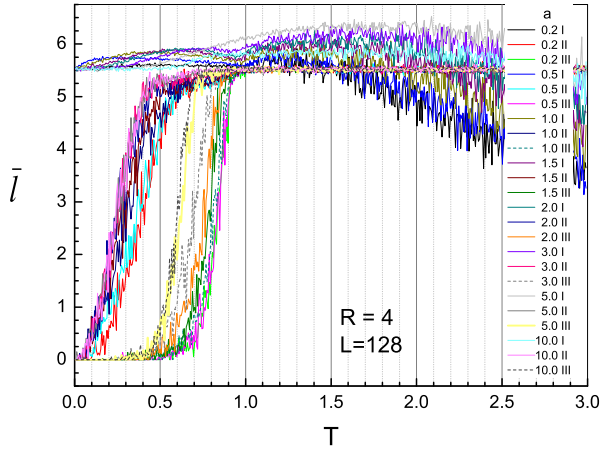


FIG. 13: (color online) Evolution of average lengths of type-I, type-II and type-III links with strategy G_{min} and probability decay exponent $a = 0.2, 0.5, 1.0, 1.5, 2.0, 3.0, 5.0$ and 10.0 in case III. $R = 4$.

any way, which ruins possible common cross point relies on the balance between F_I and F_{II} as in the 2D free space (case I of the present model) and induces the possibility to yield novel scaling relation other than any previous ones. For possible critical point, we seek help from the evolution of link fractions of 3 types. Fig.12 shows the behaviors of F_I , F_{II} and F_{III} with G_{min} for all simulated distance-decay exponents a and $R = 4$.

Correspondingly, Fig.13 shows the behaviors of \bar{l}_I , \bar{l}_{II} and \bar{l}_{III} the the same set of parameters. Critical point $T_0 = 1.0$ ($T_0 = 0.99$ to be precise) distinguishes itself from others by intuitive observation. The cross point for

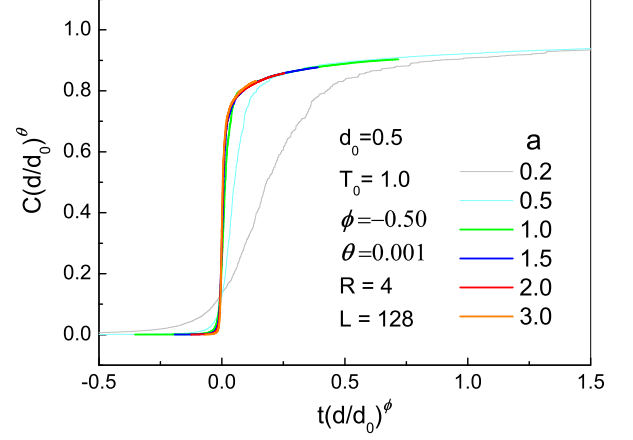


FIG. 14: (color online) Scaling of node fractions $C(T)$ with strategy G_{min} , $a = 1.0, 2.0$ and 3.0 in Case III. $R = 4$.

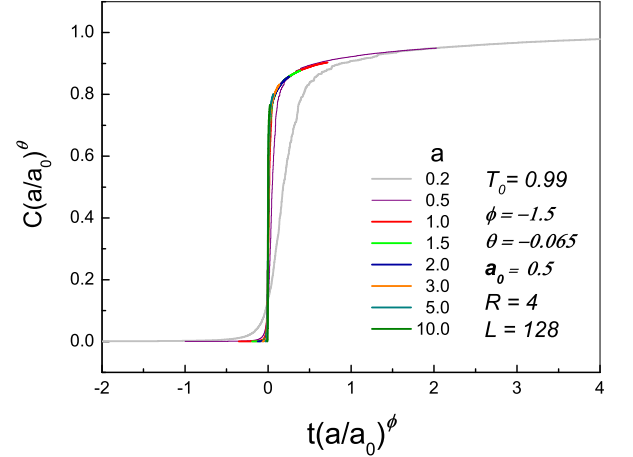


FIG. 15: (color online) Pseudo-scaling of node fractions $C(T)$ with strategy G_{min} , $a = 0.2, 0.5, 1.0, 1.5, 2.0, 3.0, 5.0$ and 10.0 in Case III. $R = 4$.

$a = 5.0$ and $a = 10.0$ go rightward from which others share, which means they could not share the same T_0 hence the same scaling relation with other decay exponents. Besides, in rescaling process for $C(T, a)$, curves for $a = 0.2$ and $a = 0.5$ failed in collapse, because smaller transmission range R inhibits the effect of slower (long-range) decay for connection probability. The rescaled function $C(t)$ for $R = 4$ is shown in Fig.14. It seems that we could go further with $\phi = -1.5$ to include more exponents a in the scaling as shown in Fig.15. However, it is meaningless in physics due to above mentioned reasons. Actually, scaling behaviors are R -dependent, but exponents ϕ and θ need not to vary. The variation of R only shifts T_0 as illustrated in Fig.16 for $R = 8$. Therefore, we

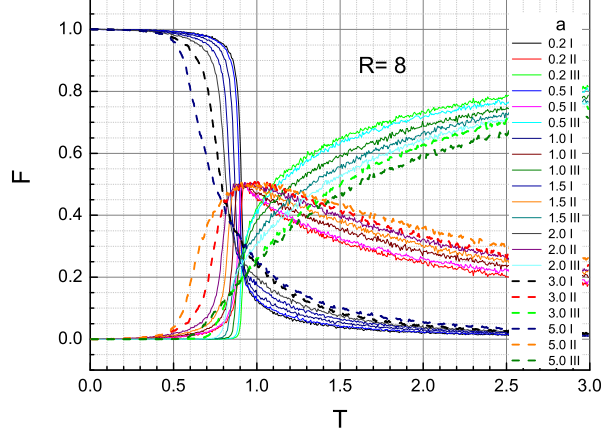


FIG. 16: (color online) Evolution of fractions of type-I, type-II and type-III links with strategy G_{min} and probability decay exponent $a = 0.2, 0.5, 1.0, 1.5, 2.0, 3.0$ and 5.0 in case III. $R = 8$.

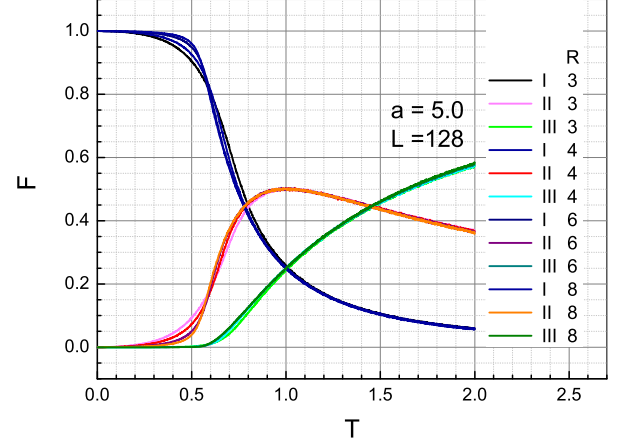


FIG. 18: (color online) Evolution of fractions of type-I, type-II and type-III links with strategy G_{max} and probability decay exponent $a = 5.0$ for $R = 3, 4, 6$ and 8 in case III.

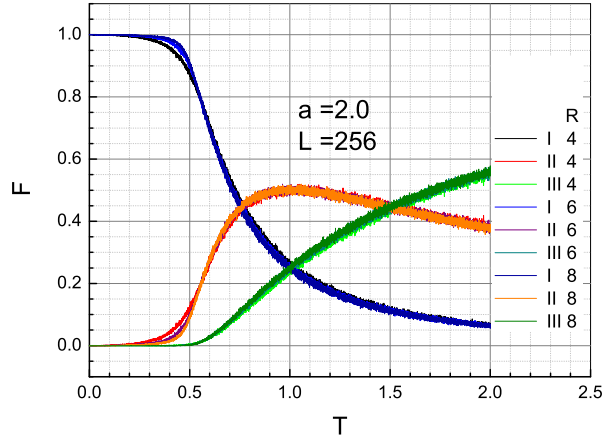


FIG. 17: (color online) Evolution of fractions of type-I, type-II and type-III links with strategy G_{max} and probability decay exponent $a = 2.0$ for $R = 4, 6$ and 8 in case III. $L = 256$. 500 realizations of network configurations.

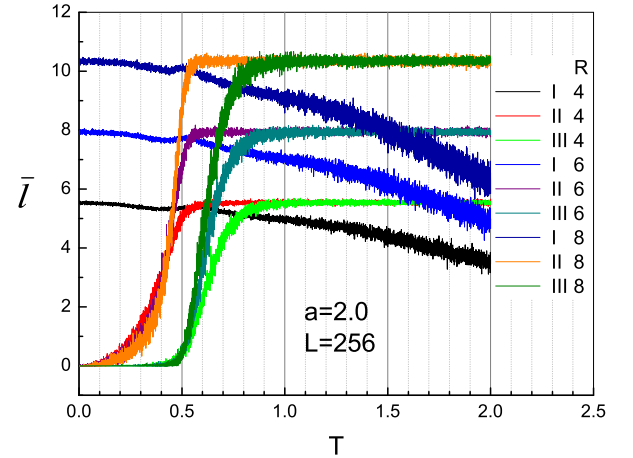


FIG. 19: (color online) Evolution of average lengths of type-I, type-II and type-III links with strategy G_{max} and probability decay exponent $a = 2.0$ for $R = 4, 6$ and 8 in case III. $L = 256$. 500 realizations of network configurations.

keep $\phi = -0.5$, $\theta = 0.005$ for all values of R with G_{min} , but take $T_0 = 0.99$ for $R = 4$, $T_0 = 0.92$ for $R = 8$, and so on. The humps above the steady level of \bar{l}_{II} and \bar{l}_{III} (all independent of exponents a) come from similar mechanism as in free 2D space but now at much lower level constrained by transmission radius R , and they are independent of size L of the system.

Scaling relation (3) for case III with strategy G_{max} inside transmission range with radius R is checked for various decay exponents a and for different sizes ($L=32, 64, 128$ and 256). Its validation is independent of size L simulated. In Fig.17 and Fig.18, the evolution of F_I ,

F_{II} and F_{III} for both $a = 2.0$ and $a = 5.0$ behave much similarly. F_I and F_{II} cross at the level a little bit lower than 0.45 , while F_I crosses F_{III} at the level 0.25 , which keeps the same as in all previous cases. The changes of exponent a and R only shift fractions along horizontal direction of figures, i.e., to change starting points and growing/dropping speed instead of levels of them. However, $T_0 = 1.0$ keeps as their common fixed point for F_I and F_{III} to cross. In Fig.19 and Fig.20, \bar{l}_I and \bar{l}_{III} for both $a = 2.0$ and $a = 5.0$ arrive at the same level hitting $T_0 = 1.0$, which distinguishes this point from totally 3 cross points, and makes up a candidate of critical point

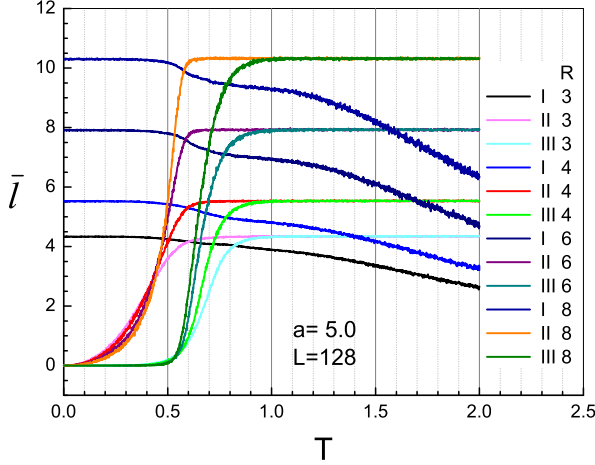


FIG. 20: (color online) Evolution of average lengths of type-I, type-II and type-III links with strategy G_{max} and probability decay exponent $a = 5.0$ for $R = 3, 4, 6$ and 8 in case III.

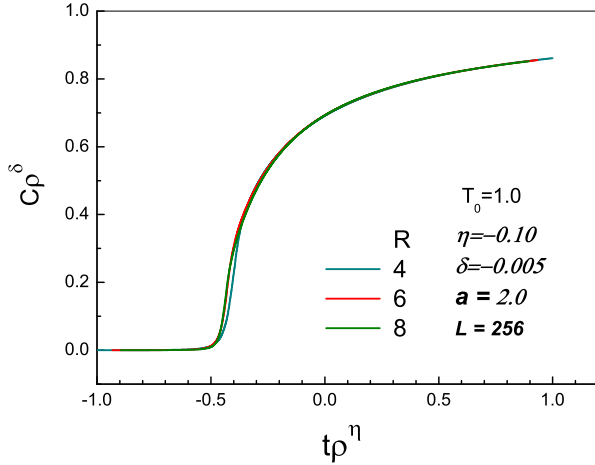


FIG. 21: (color online) Scaling of node fractions $C(T)$ with strategy G_{max} , $a = 2.0$ for $R = 4, 6$ and 8 in Case III. $L = 256$. 500 realizations of network configurations.

T_0 for scaling relations. The scaling exponents $\eta = -0.10$ and $\delta = -0.005$ have been checked for lower values of parameter a ($0.5 \leq a \leq 3.0$, Fig. 21). However, for $a = 5.0$, we have to choose a new set of exponents: $\eta = -0.25$ and $\delta = -0.01$ (Fig. 22). It is not strange that steady levels of \bar{l} keep unchanged for certain R , independent of a or L , just as that with G_{min} in case III. \bar{l} inside a circle defined by R can not go to infinity under any circumstance, but still support a critical point, which distinguishes correlated percolation in case III from case I and traditional models. It deserves further investigation.

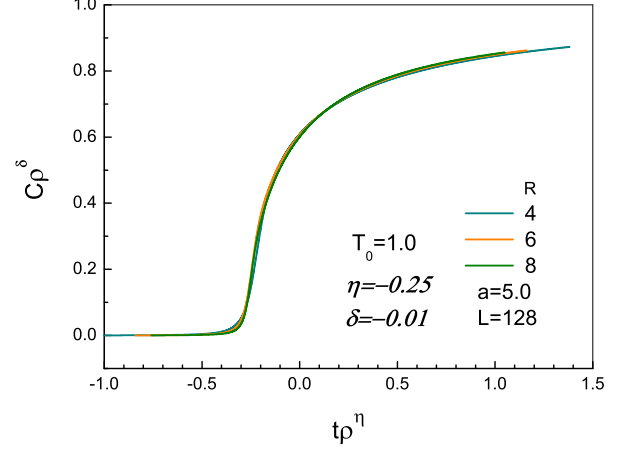


FIG. 22: (color online) Scaling of node fractions $C(T)$ with strategy G_{max} , $a = 5.0$ for $R = 3, 4, 6$ and 8 in Case III.

IV. DISCUSSION AND CONCLUSIONS

In this paper, we have proposed a new network model of correlated percolation in which geometric distance-dependent power-law decay connection probability overlaps Achlioptas process to form a gravity model. It can be tuned to facilitate or inhibit percolation with strategy G_{max} or G_{min} , cover a wide range of thresholds T_c , yield a set of new scaling relations. And it provides a scheme for better description of practical processes in complex systems.

We have developed a new approach to find out candidate critical points with physical meanings other than that of traditional ones. There are objective competition and balance between type-I and type-II, type-I and type-III links, meanwhile, competition of average lengths between type-II and type-III links. Along this line threshold T_c is found to overlap the balance point between fractions F_I and F_{II} in the explosive percolation of Achlioptas process, and the steady average lengths of three types of links are all divergent to infinity in thermodynamic limit. The percolation is indeed a transition from type-I link dominant phase to type-II and type-III dominant phase.

By observing evolutions of fractions of type-I, type-II and type-III links, a candidate critical point can be chosen combined with the message on evolutions of average lengths of them. With strategy G_{max} in 2D triangular lattice, fraction F_I get balance with F_{II} , makes up a critical point T_0 which supports scaling relation (1) in case I of the model. With strategy G_{min} inside certain transmission range with radius R , a duet balance exists for F_I and F_{III} meanwhile \bar{l}_{II} and \bar{l}_{III} , makes up another critical point T_0 which supports scaling relation (2) in case III. With strategy G_{min} and certain range $0.5 \leq a \leq 3.0$ of decay exponent a , again a duet balance exists for F_I

and F_{III} meanwhile \bar{l}_{II} and \bar{l}_{III} , makes up another critical point T_0 which supports scaling relation (3) for a mini-scale of R in case III. This approach serves an assistant tool in seeking critical points of order parameter $C(T)$ which is usually not easy to determine in an intuitive way.

In numerical calculations, besides percolation threshold T_c , two fixed points, $T_0 = 0.78$ and $T_0 = 1.0$ emerge as distinct points not only for special temporal crux but also for unchanged levels of F_I , F_{II} and F_{III} inherited from AP, which is expected to be further proved in analytical ways. However, they have different physical meanings. The former corresponds to a divergent average length of links, while the later corresponds to confined average lengths by transmission range R , which distinguishes itself from traditional critical points in percolations.

Correlated percolations are relevant since long-range correlation drastically changes the critical properties. The validation ranges of decay exponents a with various strategies in different cases define the relevance of correlation. They have demonstrated novel scaling relations different from traditional 2D short-range percolation in triangular lattice. The intervention of distance-dependent power-law decay ingredients alleviates the explosive effect of percolation transition by horizontal adjustment of evolutions along temporal axis, separates T_0

from T_c , while the overlapped AP included in the present gravity model always conquers the vertical levels of three fractions and average lengths, which are found neither in traditional correlated percolations of continuities in 2D space nor in complex networks. Moreover, the node fraction $C(T)$ of the largest component, fractions $F(T)$, and average lengths $\bar{l}(T)$ of three types of links all show saturation phenomena as pointed out by Weinrib but with different values of exponent of a since now AP overlaps in the present gravity model. And scaling law of Weinrib is no longer obeyed according to the evidence of numerical results of average-length exponents.

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- [1] A. Coniglio, H. E. Stanley and C. Klein, Phys. Rev. Lett. **42**, 518 (1979).
 - [2] A. Coniglio and T. C. Lubensky, J. Phys. A: Math. Gen. **13** (1980) 1783-1789.
 - [3] G. F. Tuthillt and W. Klein, J. Phys. A: Math. Gen. **15** (1982) L377-L384.
 - [4] J. Benzoni and J. L. Cardy, J. Phys. A: Math. Gen. **17** (1984) 179-196.
 - [5] P. J. M. Bastiaansen and H. J. F. Knops, J. Phys. A: Math. Gen. **30** (1997) 1791-1801.
 - [6] Hernán A. Makse, S. Havlin, M. Schwartz and H. E. Stanley, Arxiv./Cond-matt/9512024.
 - [7] Chin-Kun Hu, Phys. Rev. B **29**, 5103 (1984).
 - [8] H. A. Makse, J. S. Andrade, Jr., Michael Batty, S. Havlin, and H. E. Stanley, Phys. Rev. E **58**, 7054(1998).
 - [9] C. M. Chaves, B. Koiller, Physica A **218**, 271(1995).
 - [10] H. Yang, W. Zhang, N. Bray-Ali and S. Haas, Arxiv.Cond-mat/0908.0104.
 - [11] S. D. S. Reis, A. A. Moreira and J. S. Andrade Jr. 1112.0557.
 - [12] J. østein, H. Bakke, and J. Bjelland, Physica Scripta. T106, 65-69 (2003).
 - [13] H. A. Schuh, R. W. Minich and M. Kumary Phil. Mag., **83**, 711-726 (2003).
 - [14] A. D. Araújo, A. A. Moreira, R. N. C. Filho, and J. S. Andrade, Jr. Phys. Rev. E **67**, 027102(2003).
 - [15] A. Weinrib and B.I. Halperin, Phys. Rev. B **27**, 413(1983).
 - [16] Abel Weinrib, Phys. Rev. B **29**, 387 (1984).
 - [17] A. B. Harris, J. Phys. C, **7**, 1671(1974).
 - [18] D. Achlioptas, R. M. D'Souza, and J. Spencer, Science **323**, 1453 (2009).
 - [19] R. M. Ziff, Phys. Rev. Lett. **103**, 045701 (2009).
 - [20] Y. S. Cho, *et al.*, Phys. Rev. Lett. **103**, 135702 (2009).
 - [21] F. Radicchi and S. Fortunato, Phys. Rev. Lett. **103**, 168701 (2009).
 - [22] E. J. Friedman and A. S. Landsberg, Phys. Rev. Lett. **103**, 255701 (2009).
 - [23] R. M. D'Souza and M. Mitzenmacher, Phys. Rev. Lett. **104**, 195702 (2010).
 - [24] N. A. M. Araujo and H. J. Herrmann, Phys. Rev. Lett. **105**, 035701 (2010).
 - [25] Y. S. Cho, *et al.*, Phys. Rev. E **82**, 042102 (2010).
 - [26] A. Moreira *et al.*, Phys. Rev. E **81**, 040101 (2010).
 - [27] Y. Kim *et al.*, Phys. Rev. E **82**, 061105 (2010).
 - [28] Y. S. Cho, *et al.*, Phys. Rev. E **81**, 030103 (2010).
 - [29] F. Radicchi *et al.*, Phys. Rev. E **81**, 036110 (2010).
 - [30] N. Araújo *et al.*, Phys. Rev. Lett. **106**, 095703 (2011).
 - [31] J. S. Andrade Jr. *et al.*, Phys. Rev. E **83**, 031133 (2011).
 - [32] J. Nagler *et al.*, Nat. Phys. **7**, 265 (2011).
 - [33] Y. S. Cho and B. Kahng, Phys. Rev. Lett. **107**, 275703 (2011).
 - [34] R. A. da Costa *et al.*, Phys. Rev. Lett. **105**, 255701 (2010).
 - [35] O. Riordan and L. Warnke, Science **333**, 322(2011).
 - [36] H. K. Lee *et al.*, Phys. Rev. E **84**, 020101(R) (2011).
 - [37] P. Grassberger, C. Christensen, G. Bizhani, S.-W. Son, and M. Paczuski, Phys. Rev. Lett. **106**, 225701 (2011).
 - [38] L. Tian and D.-N. Shi, Phys. Lett. A **391**, 1234 (2012).
 - [39] C. -P. Zhu *et al.*, to be published.
 - [40] C. Moore and M. E. J. Newman, Phys. Rev. E **61**,

- 5678(2000); D. S. Callaway, *et al.*, Phys. Rev. Lett. **85**, 5468(2000).
- [41] C. E. Perkins, *Ad Hoc Networking*. (Addison-Wesley, New York, 2000).
- [42] L. Wang *et al.*, Phys. Rev. E **78**, 066107 (2008).
- [43] J. Tinbergen, Twentieth Century Fund, New York (1962); P. Poyhonen, Weltwirtschaftliches Archiv, 93 (1963); J. E. Anderson, The American Economic Review, 69, 106(1979). J. H. Bergstrand, The review of economics and statistics.1985. A. V. Deardorff, The Regionalization of the World Economy, Chicago Univ. Press, Chicago, USA (1998). E. Helpman, J. Eco. Lit., 44, 589(2006).
- [44] G. Li *et al.*, Phys. Rev. Lett. **104**, 018701 (2010).
- [45] R. R. Roy, *Handbook of Mobile Ad Hoc Networks for Mobility Models Part 2*, pp. 443-482 (2011).
- [46] M. Nekovee, New J. Phys., **9**,189(2007).

